Dispersion relations for thermally excited waves in plasma crystals

2003 J. Phys. A: Math. Gen. 36 6109

(http://iopscience.iop.org/0305-4470/36/22/337)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) 6109–6115 PII: S0305-4470(03)55465-8

Dispersion relations for thermally excited waves in plasma crystals

K Qiao and T W Hyde

Center for Astrophysics, Space Physics and Engineering Research, Baylor University, PO Box 97310, Waco, TX 76798-7310, USA

Received 30 October 2002, in final form 15 January 2003
Published 22 May 2003
Online at stacks.iop.org/JPhysA/36/6109

Abstract

Thermally excited waves in a plasma crystal were numerically simulated using a Box_Tree code. The code is a Barnes_Hut tree code proved effective in modelling systems composed of a large number of particles. Interaction between individual particles was assumed to conform to a Yukawa potential. Particle charge, mass, density, Debye length and output data intervals are all adjustable parameters in the code. Employing a Fourier transform on the output data, dispersion relations for both longitudinal and transverse wave modes were determined. These were compared with the dispersion relations obtained from experiment as well as a theory based on a harmonic approximation to the potential. They were found to agree over a range of $0.9 < \kappa < 5$, where κ is the shielding parameter and is defined as the ratio between interparticle distance a and dust Debye length λ_D. This is an improvement over experimental data as current experiments can only verify the theory up to $\kappa = 1.5$.

PACS numbers: 52.27.Lw, 25.52.2b

1. Introduction

There has been an increasing interest in the field of complex plasmas for the past two decades, especially after Coulomb crystals were first experimentally observed at various laboratories [1–3]. Recently, the study of waves in plasma crystals has become one of the more active research fields, since as in condensed matter systems, wave propagation is a research topic with both theoretical and practical values.

Experimental research has been conducted [4–8] on both longitudinal and transverse waves in 2D Coulomb crystals. In these experiments, a 2D hexagonal lattice of dust particles was established within the sheath region of the lower electrode in a rf discharge plasma. Dust particles introduced into the plasma become charged and are subject to mutual Coulomb interactions. The particles interact with each other through a Yukawa potential, $V(r) = (Q/r)\exp(-r/\lambda_D)$, where λ_D is the Debye length, which helps determine the manner in which the charge will be shielded by the plasma. The particles are levitated by an electric field in the

sheath region and form a 2D Coulomb crystal. The Coulomb crystal is primarily characterized by two parameters, the shielding parameter $\kappa = a/\lambda_D$ and the Coulomb coupling parameter $\Gamma = Q^2/4\pi\varepsilon_0 a k_B T$, where a is the mean interparticle spacing and T is the kinetic temperature of the particles.

Several laboratory experiments have determined that exciting particles in a shaped region [4–7] of the lattice can produce both longitudinal and transverse waves. In these experiments, particle excitation was created using a sinusoidally modulated laser beam. Under the radiation pressure created by the laser, the particles within the excitation region oscillate sinusoidally, and a wave propagates through the lattice. Fourier analysis of the subsequent particle motion then allows the dispersion relations to be determined. More recently, experiments [8] examining waves created via thermal excitation have been conducted. The dispersion relations for both the longitudinal and transverse waves were again determined employing Fourier analysis of the spontaneous motion of the particles within the lattice. Comparison of the results of the two experiments shows that they agree with one another, even though the results from thermally produced waves cover a larger region in k space.

A theoretical model has recently been presented [9] to explain these experimental results. Wang *et al* analytically derived the dispersion relations for both the longitudinal and transverse waves on a 2D hexagonal lattice assuming a harmonic approximation for the motion of the particles about their equilibrium positions and a Yukawa interparticle potential. Wang *et al* showed that not only do that the dispersion relations depend upon the direction of propagation but also waves propagating parallel to the primitive translation vectors have different dispersion relations than those propagating perpendicular to the primitive translation vectors. In both cases, the dispersion relations were shown to agree with experiment.

2. Methods

In this research, a Box_Tree code was employed to simulate the spontaneous thermal wave in a Coulomb crystal. The results were Fourier analysed to obtain the dispersion relations and then compared with both the theoretical results discussed above and experimental data.

The Box_Tree code is a Barnes-Hut tree code first written by Richardson [10] for planetary ring and disk studies. It was later modified by Matthews [11] to include electrostatic interactions and then by Vasut [12] to simulate the formation of plasma crystals. Box_Tree is proving to be an effective tool for modelling real-time systems, particularly those composed of a large number of particles with specific interparticle interactions.

The Box_Tree code models a dusty plasma system by first dividing it into self-similar patches, where the box size is much greater than the radial mean excursions of the constituent dust grains and the boundary conditions are met using 26 ghost boxes. A tree code is incorporated into the routine to allow it to deal with interactions between the particles. Interparticle interactions are calculated by examining the multipole expansions of collections of particles, so the code scales as $N \log N$ instead of N^2. Data files showing each particle's position and velocity are output for analysis once a certain user specified time interval is reached. Particle charge, mass, density, average initial velocity, Debye length and output data intervals are all adjustable parameters within the code.

All calculated interparticle interactions include both gravitational and electrostatic forces. The interparticle electrostatic force is assumed to be a 3D screened Coulomb repulsion or Yukawa potential. The simulation box is a 3D box and all calculations are handled in a 3D manner. However, the initial positions of the particles are constrained to lie in a horizontal plane (as is often the case in an experimental setting) and their initial velocities are specified to contain only XY components. The gravitational force from the earth and the electrostatic

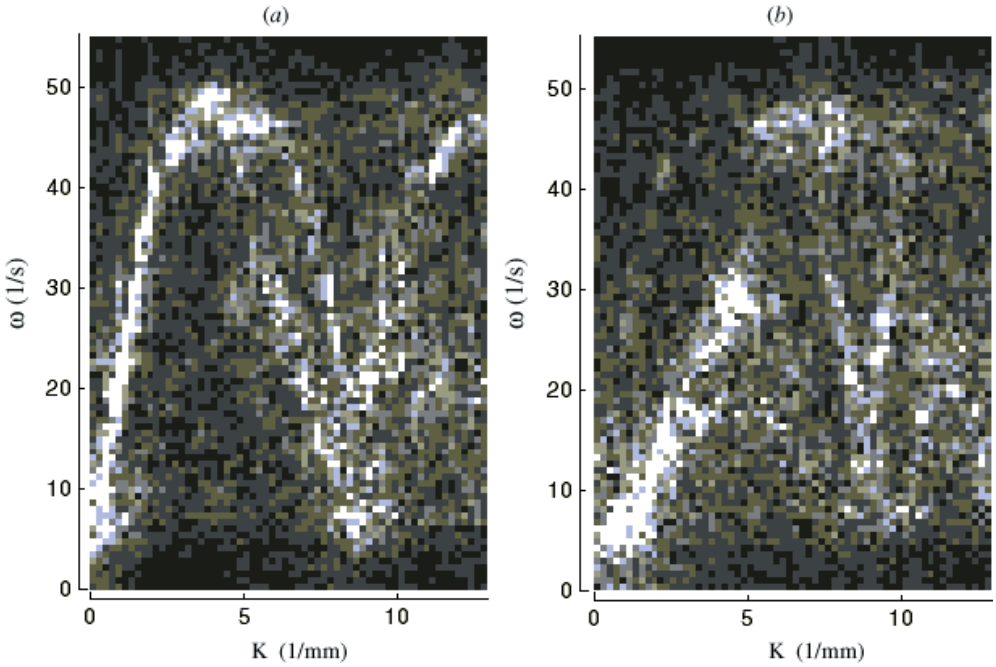

Figure 1. Dispersion relations from the simulation of (*a*) longitudinal and (*b*) transverse waves with an arbitrary propagation direction in a Coulomb crystal with $\kappa = 1.4$ ($\lambda_D = 570$ μm).

force produced by the (lower) powered electrode are assumed to balance one another and are neglected. Therefore, the particles are restricted to a 2D plane (within the 3D box) throughout the simulation since they are not acted upon by forces (or velocities) in the Z direction.

Initial conditions include a random distribution of 679 particles in a $20 \times 20 \times 20$ mm^3 box. The neutral gas drag is included with a specified Epstein drag coefficient of $\gamma = 1.22$. After about 20 s, the state of the system was determined to be in a crystalline form (solid) using the pair correlation function. The pair correlation function was employed (along with ordering and Voronoi diagrams) instead of the usual phase space diagrams often seen in MD simulations since the kinetic energy (temperature) of the particles will continue to decrease throughout the simulation due to gas drag. This manifests itself in a cooling of the system making the usual phase space diagrams difficult to use due to their time dependence [12]. In all of the simulations the diameter of the embedded particles is 6.5 μm, the density is 1.51 g cm^{-3}, the charge is 14500 e and the interparticle distance (as determined from the pair correlation function) is around 825 μm.

Once the Coulomb lattice is established, the random thermal motion of the particles within this lattice is tracked for 10 s by choosing the data output time interval to be 0.01 s and collecting 1000 data files. Depending on the equilibrium particle position, the particles are divided into bins with the particle velocity averaged over each bin, yielding velocity data, which is dependent upon position. Combining data files, a velocity matrix depending on time and position can then be obtained. A double Fourier transformation of this matrix (as given below) yields a matrix representing the particle velocity in k–ω space.

$$V_{k,\omega} = 2/TL \int_0^T \int_0^L v(x,t) \exp\left[-\int \mathrm{i}(kx - \omega t)\right] \mathrm{d}x\, \mathrm{d}t.$$

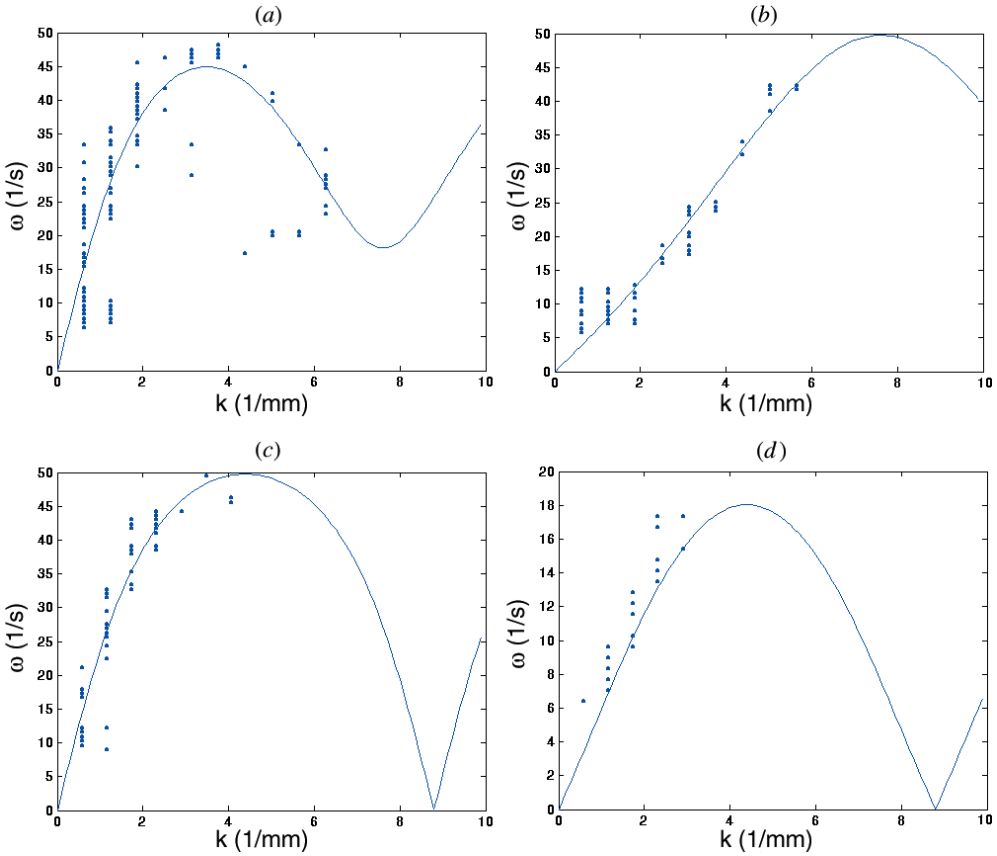

Figure 2. Comparison of dispersion relations as obtained from simulation and Wang's theory. Dots represent data obtained from simulation. Solid lines are dispersion relations from Wang's theory. Figures 2(*a*) and (*b*) are for waves propagating parallel to the primitive translation vector while (*c*) and (*d*) are for waves propagating perpendicular to the primitive translation vector. Additionally, (*a*) and (*c*) are longitudinal waves, while (*b*) and (*d*) are transverse waves. In all cases, $\kappa = 1.4$ ($\lambda_D = 570\ \mu$m).

The number of particles (or the box size) will limit the resolution of any subsequent graphs but the minimum wavelength is determined by the width of the bins chosen to analyse the data. For best results, the bin width must be comparable to the minimum interparticle spacing along the specified propagation direction. Waves with wavelengths shorter than the minimum interparticle spacing are not detectable.

The possibility of temperature fluctuations of the system (temperature control/system thermostat) is carefully monitored and has been shown to be negligible for all conditions examined by this simulation. (See [12] for complete details of this method). The dispersion relations were also examined for various temperatures to ensure that they did not depend on the overall system temperature.

3. Results

Figure 1 shows the k–ω space velocity matrix obtained by using the equation given above (for an arbitrary propagation direction) employing pixel brightness corresponding to velocity

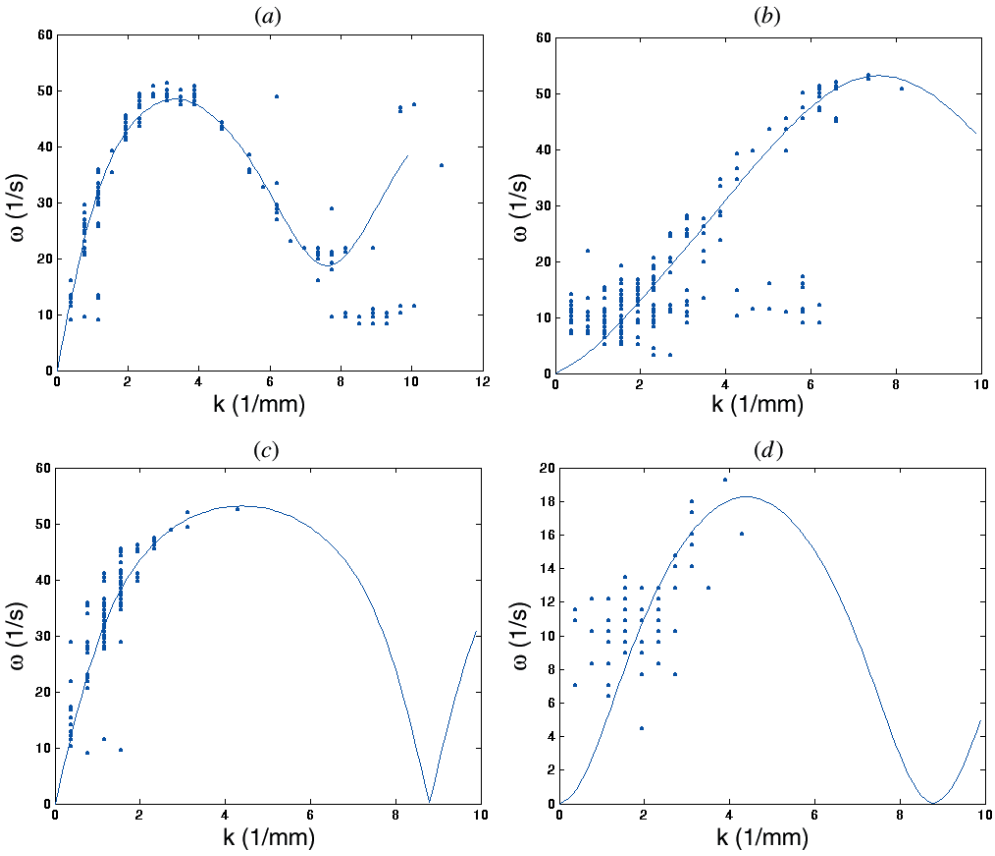

Figure 3. Same as for figure 2 but with $\kappa = 0.9$ ($\lambda_D = 0.92$ mm).

value. Bright areas show the dispersion relation of the (*a*) longitudinal and (*b*) transverse spontaneous waves for $\kappa = 1.4$ ($\lambda_D = 570$ μm) and particle charge $q = 1.92 \times 10^{-15}$ C. To obtain the dispersion relation for a particular wave mode (longitudinal or transverse), velocity components perpendicular to the bins for longitudinal waves and parallel to the bins for transverse waves were used.

In order to obtain the dispersion relation of waves with a specific propagation direction, the bins were chosen to be perpendicular to the specified direction. Accordingly, the dispersion relations shown by data points in figure 2 were obtained by establishing minimum intensity level in the k–ω space velocity matrix generated by specifying a propagation direction as described above. Superimposing the dispersion relation obtained from Wang's theory as a solid line, it can be seen that the raw simulation data are a solid fit. As mentioned above, the possible wavelengths observed are limited by the width of the bins, which in turn is limited by the interparticle spacing. Thus, the maximum wave number k is about 4 mm^{-1} for propagation perpendicular to the primitive translation vector and 6 mm^{-1} for propagation parallel to the primitive translation vector. The fact that the Coulomb crystal modelled is not an ideal solid lattice is responsible for the data points for waves propagating perpendicular to the primitive translation vector, sometimes agreeing with the theoretical dispersion relation of waves propagating parallel to the primitive vector (and vice versa) as can be seen in figure 3(*b*).

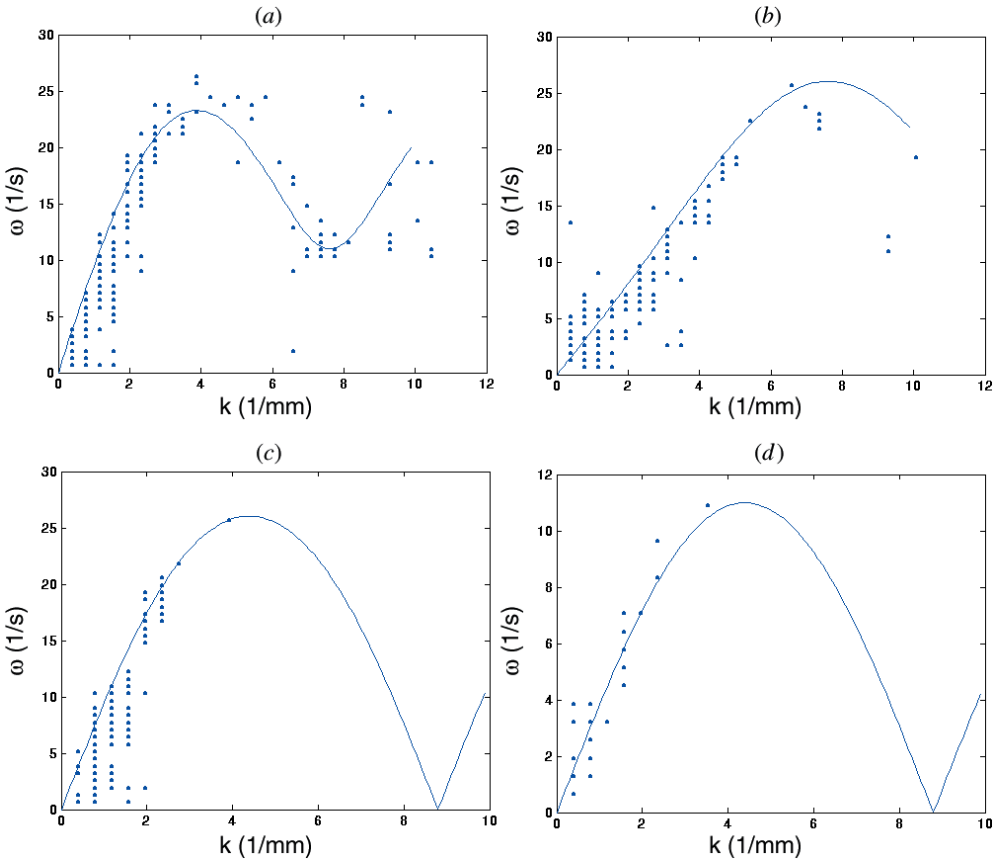

Figure 4. Same as for figure 2 but with $\kappa = 5$ ($\lambda_D = 0.17$ mm).

The simulations described above were run for different average initial velocities ranging from 0 to 1.0×10^{-3} m s^{-1}. The dispersion relations were seen to be almost identical, showing them to be temperature independent. Simulations were also run for different κ values, ranging over the regime $0.9 < \kappa < 5$. The results for $\kappa = 0.9$ ($\lambda_D = 0.92$ mm) and $\kappa = 5$ ($\lambda_D = 0.17$ mm) are shown in figures 3 and 4. Dispersion relations obtained from the raw data were again found to be in agreement with the theory for both longitudinal and transverse waves. For values of $\kappa > 5$, the dispersion is no longer clear enough to obtain distinguishable data.

4. Conclusions

As can be seen, Box_Tree produces dispersion relations which are in good agreement with current theory. Since Box_Tree can easily simulate a larger range of κ than is experimentally possible, this allows it to be used as an effective numerical tool for simulation of various Yukawa systems. Additionally, system parameters are much easier to adjust within Box_Tree than in an experimental setting and they can cover regimes (such as high frequencies) which are difficult (if not impossible) to reach physically.

Finally, the code is not predicated on the assumption of a harmonic approximation and/or lattice structure. The only assumptions required by the program are a Yukawa potential and

the specification of constant parameters (i.e. charge, mass, Debye length, etc) by the user. This should allow Box_Tree to become an effective tool in simulating nonlinear effects or complex plasma liquids.

References

[1] Chu J H and Lin I 1994 *Phys. Rev. Lett.* **72** 4009
[2] Hayashi Y and Tachibana K 1994 *Japan. J. Appl. Phys.* **33** L804
[3] Thomas H and Morfill G E 1994 *Phys. Rev. Lett.* **73** 652–5
[4] Homann A *et al* 1997 *Phys. Rev.* E **56** 7138
[5] Homann A *et al* 1998 *Phys. Lett.* A **242** 173
[6] Nunomura S *et al* 2000 *Phys. Rev. Lett.* **84** 5141
[7] Nunomura S *et al* 2002 *Phys. Rev.* E **65** 66402
[8] Nunomura S *et al* 2002 *Phys. Rev. Lett.* **89** 35001
[9] Wang Xiaogang *et al* 2001 *Phys. Rev. Lett.* **86** 2569
[10] Richardson D 1994 *Mon. Not. R. Astron. Soc.* **269** 493–511
[11] Matthews L 1998 *PhD Dissertation* Baylor University
[12] Vasut J and Hyde T W 2001 *IEEE Trans. Plasma Sci.* **29** 231–7